Chapter 1: Preliminaries.

0.1. The real numbers.

Example 1.1. Solve the linear inequality $-3x+5<-13$.

Solution. Using the properties of real numbers, we have

$$-3x+5<-13$$

$$-3x+5-5<-13-5$$

$$-3x<-18$$

$$\frac{-3x}{-3}>\frac{-18}{-3}$$

$$x>6$$

The solution set is $(6,\infty)=\{x\in\mathbb{R}|x>6\}$.

Example 1.2 Solve the inequality $11>5-3x\geq-13$.

Solution. Using the properties of real numbers, we have

$$11>5-3x\geq-13$$

$$11-5>5-5-3x\geq-13-5$$

$$6>-3x\geq-18$$

$$-2<x\leq6$$

The solution set is $(-2,6]=\{x\in\mathbb{R}|-2<x\leq6\}$.

Example 1.3 Solve the inequality $x^2-5x+6>0$.

Solution. The factoring of $x^2-5x+6>0$ is

$$(x-3)(x-2)>0$$

	$(-\infty,2)$	$(2,3)$	$(3,\infty)$
$(x-3)$	$---$	$---$	$+++$
$(x-2)$	$---$	$+++$	$+++$
$x^2-5x+6=(x-3)(x-2)$	$+++$	$---$	$+++$

The solution set is $(-\infty,2)\cup(3,\infty)$.

Example 1.4. Solve the inequality $\dfrac{x-1}{x+2}\geq0$.

Solution. We have $x\neq-2$, and

	$(-\infty,-2)$	$(-2,1)$	$(1,\infty)$
$(x-1)$	$---$	$---$	$+++$
$(x+2)$	$---$	$+++$	$+++$
$\dfrac{x-1}{x+2}$	$+++$	$---$	$+++$

The solution set is $(-\infty,-2)\bigcup[1,\infty)$.

Example 1.5. Solve the inequality $|x-3|<4$.

Solution. We use the property $|x|<k \Rightarrow -k<x<k$. Then

$$|x-3|<4$$
$$\Rightarrow -4<x-3<4$$
$$\Rightarrow -4+3<x-3+3<4+3$$
$$\Rightarrow -1<x<7$$

The solution set is $(-1,7)=\{x\in\mathbb{R}\,|\,-1<x<7\}$.

If $|x-3|\le 4$, then we use $|x|\le k \Rightarrow -k\le x\le k$. Thus

$$|x-3|\le 4$$
$$\Rightarrow -4\le x-3\le 4$$
$$\Rightarrow -4+3\le x-3+3\le 4+3$$
$$\Rightarrow -1\le x\le 7$$

solution set is $[-1,7]=\{x\in\mathbb{R}\,|\,-1\le x\le 7\}$.

Example 1.6. Solve the inequality $|x-3|>4$.

Solution. We use the property $|x|>k \Rightarrow -k>x$ or $x>k$ Then

$$|x-3|>4$$
$$\Rightarrow -4>x-3 \text{ or } x-3>4$$
$$\Rightarrow -4+3>x-3+3 \text{ or } x-3+3>4+3$$
$$\Rightarrow -1>x \text{ or } x>7$$

The solution set is $(-\infty,-1)\bigcup(7,\infty)$.

If $|x-3|\ge 4$, then we use $|x|\ge k \Rightarrow -k\ge x$ or $x\ge k$. Then

$$|x-3| \geq 4$$

$$\Rightarrow -4 \geq x-3 \text{ or } x-3 \geq 4$$

$$\Rightarrow -4+3 \geq x-3+3 \text{ or } x-3+3 \geq 4+3$$

$$\Rightarrow -1 \geq x \text{ or } x \geq 7$$

The solution set is $(-\infty,-1]\cup[7,\infty)$.

Example 1.7. Find the distance between the points $(-2,-5)$ and $(3,1)$.

Solution. The distance between the points (x_1,y_1) and (x_2,y_2) is

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Then

$$\begin{aligned} d &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(3-(-2))^2+(1-(-5))^2} \\ &= \sqrt{(3+2)^2+(1+5)^2} \\ &= \sqrt{5^2+6^2} \\ &= \sqrt{25+36} \\ &= \sqrt{61} \end{aligned}$$

Example 1.7. Find the distance between the pairs $(1,2),(3,4)$ and $(2,6)$. Use the distances to determine if the points forms vertices of a right triangle.

Solution.

$$\begin{aligned} d((1,2),(3,4)) &= \sqrt{(3-1)^2+(4-2)^2} \\ &= \sqrt{4+4} \\ &= \sqrt{8} \end{aligned}$$

$$\begin{aligned} d((1,2),(2,6)) &= \sqrt{(2-1)^2+(6-2)^2} \\ &= \sqrt{1+16} \\ &= \sqrt{17} \end{aligned}$$

$$d((3,4),(2,6)) = \sqrt{(2-3)^2 + (6-4)^2}$$
$$= \sqrt{1+4}$$
$$= \sqrt{5}$$

The sides of the right triangle must satisfy the Pythagorean Theorem, which require that $(\sqrt{8})^2 + (\sqrt{5})^2 = (\sqrt{17})^2$ which is incorrect. Thus the triangle is not a right triangle.

Exercises 0.1

I) Solve the inequality.

1) $3x + 2 < 11$

Sol: $x < 3 \Rightarrow (-\infty, 3)$

2) $4 - 3x < 6$

Sol: $x < -\frac{3}{2} \Rightarrow (-\infty, -\frac{3}{2})$

3) $4 \le x + 1 < 7$

Sol: $3 \le x < 6 \Rightarrow [3,6)$

4) $x^2 + 3x - 4 > 0$

Sol: $x < -4$ or $x > 1 \Rightarrow (-\infty, -4) \cup (1, \infty)$

5) $|3 - x| < 1$

Sol: $2 < x < 4 \Rightarrow (2,4)$

6) $|2x + 1| > 2$

Sol: $-\frac{3}{2} > x$ or $x > \frac{1}{2} \Rightarrow (-\infty, -\frac{3}{2}) \cup (\frac{1}{2}, \infty)$

7) $\frac{x+2}{x-2} > 0$

Sol: $-2 > x$ or $x > 2 \Rightarrow (-\infty, -2) \cup (2, \infty)$

8) $\frac{x^2 - x - 2}{(x+4)^2} > 0$

Sol:

$-4 > x$ or $-4 < x < -1$ or $x > 2 \Rightarrow (-\infty, -4) \cup (-4, -1) \cup (2, \infty)$

II) Find the distance between the pair of points

9) $(2,1)$ & $(4,4)$

Sol: $\sqrt{13}$

10) $(-1,-2)\ \&\ (3,-2)$
Sol: 4
11) $(0,2)\ \&\ (-2,6)$
Sol: $\sqrt{20}$
III) Determine if the set of points forms the vertices of a right triangle.
12) $(1,1)\ \&\ (3,4)\ \&\ (0,6)$
Sol: Yes
13) $(-2,3)\ \&\ (2,9)\ \&\ (-4,13)$
Sol: Yes

King Abdul Aziz University Mathematics Department Math 110 Workshop 1: Real Numbers.
1) The whole number in $\mathbb{W}$ is $\boxed{A}$ $\sqrt[3]{8}$ $\boxed{B}$ -12 $\boxed{C}$ 5.3 $\boxed{D}$ $\frac{2}{3}$
2) The whole number in $\mathbb{W}$ is $\boxed{A}$ $-\sqrt[3]{8}$ $\boxed{B}$ 12 $\boxed{C}$ 0.5 $\boxed{D}$ π
3) The whole number in $\mathbb{W}$ is $\boxed{A}$ -2 $\boxed{B}$ π $\boxed{C}$ 0 $\boxed{D}$ -3.2
4) The whole number in $\mathbb{W}$ is $\boxed{A}$ -2 $\boxed{B}$ π $\boxed{C}$ -3.2 $\boxed{D}$ $\sqrt{25}$
5) The integer in $\mathbb{Z}$ is $\boxed{A}$ $\sqrt{25}$ $\boxed{B}$ $\sqrt{-2}$ $\boxed{C}$ 5.3 $\boxed{D}$ $\frac{2}{3}$
6) The integer in $\mathbb{Z}$ is $\boxed{A}$ π $\boxed{B}$ 12 $\boxed{C}$ 5.3 $\boxed{D}$ -3.2
7) The integer in $\mathbb{Z}$ is $\boxed{A}$ π $\boxed{B}$ $\sqrt{-2}$ $\boxed{C}$ 5.3 $\boxed{D}$ 0
8) The integer in $\mathbb{Z}$ is $\boxed{A}$ π $\boxed{B}$ $\sqrt{-2}$ $\boxed{C}$ $-\sqrt[3]{8}$ $\boxed{D}$ 5.3
9) The irrational in $\mathbb{I}$ is $\boxed{A}$ 12 $\boxed{B}$ $\sqrt{-2}$ $\boxed{C}$ $\sqrt[3]{4}$ $\boxed{D}$ 0
10) The irrational in $\mathbb{I}$ is $\boxed{A}$ $\frac{2}{3}$ $\boxed{B}$ $\sqrt{-2}$ $\boxed{C}$ 0 $\boxed{D}$ $\sqrt[5]{5}$

11) The rational in $\mathbb{Q}$ is

[A] $\frac{2}{3}$ [B] $\sqrt{-2}$ [C] $\sqrt[3]{4}$ [D] $\sqrt[5]{5}$

12) The rational in $\mathbb{Q}$ is

[A] $\sqrt[5]{5}$ [B] $-\sqrt{2}$ [C] $\sqrt[3]{4}$ [D] $\sqrt{25}$

13) The rational in $\mathbb{Q}$ is

[A] $\sqrt[5]{5}$ [B] $4\frac{2}{3}$ [C] $\sqrt[3]{4}$ [D] $-\sqrt{2}$

14) The natural number in $\mathbb{N}$ is

[A] 4 [B] $4\frac{2}{3}$ [C] $\sqrt[3]{4}$ [D] -12

15) The natural number in $\mathbb{N}$ is

[A] $\sqrt[3]{125}$ [B] 0 [C] $\sqrt[3]{4}$ [D] -12

16) The real number in $\mathbb{R}$ is

[A] $-\sqrt{-2}$ [B] $\sqrt{-1}$ [C] $\sqrt[4]{-8}$ [D] $-\sqrt{49}$

17) $\{x \in \mathbb{R} \mid -3 \le x \le 3\} =$

[A] $[-3,3]$ [B] $(-3,3)$ [C] $(-3,3]$ [D] $[-3,3)$

18) $\{x \in \mathbb{R} \mid -2 < x < 5\} =$

[A] $[-2,5]$ [B] $(-2,5)$ [C] $(-2,5]$ [D] $[-2,5)$

19) $\{x \in \mathbb{R} \mid -2 \le x < 5\} =$

[A] $[-2,5]$ [B] $(-2,5)$ [C] $(-2,5]$ [D] $[-2,5)$

20) $\{x \in \mathbb{R} \mid -2 < x \le 5\} =$

[A] $[-2,5]$ [B] $(-2,5)$ [C] $(-2,5]$ [D] $[-2,5)$

21) $\{x \in \mathbb{R} \mid x \le -2\} =$

[A] $(-\infty,-2]$ [B] $(-\infty,-2)$ [C] $(-2,\infty)$ [D] $[-2,\infty)$

22) $\{x \in \mathbb{R} \mid x \ge -2\} =$

[A] $(-\infty,-2]$ [B] $(-\infty,-2)$

[C] $(-2,\infty)$ [D] $[-2,\infty)$

23) $\{x \in \mathbb{R} \mid x < -2\} =$

[A] $(-\infty,-2]$ [B] $(-\infty,-2)$ [C] $(-2,\infty)$ [D] $[-2,\infty)$

24) $\{x \in \mathbb{R} | x > -2\} =$

[A] $(-\infty, -2]$ [B] $(-\infty, -2)$ [C] $(-2, \infty)$ [D] $[-2, \infty)$

25) $6 \notin$

[A] $\mathbb{R}$ [B] $\mathbb{N}$ [C] $\mathbb{Q}$ [D] $\mathbb{I}$

26) $3.2 \in$

[A] $\mathbb{Z}$ [B] $\mathbb{N}$ [C] $\mathbb{Q}$ [D] $\mathbb{I}$

27) $-\sqrt{6} \in$

[A] $\mathbb{W}$ [B] $\mathbb{N}$ [C] $\mathbb{Q}$ [D] $\mathbb{I}$

28) $\mathbb{Q} \subset$

[A] $\mathbb{R}$ [B] $\mathbb{N}$ [C] $\mathbb{W}$ [D] $\mathbb{I}$

29) $\mathbb{I} \subset$

[A] $\mathbb{Q}$ [B] $\mathbb{N}$ [C] $\mathbb{W}$ [D] $\mathbb{R}$

30) $\mathbb{W} \subset$

[A] $\varnothing$ [B] $\mathbb{N}$ [C] $\mathbb{I}$ [D] $\mathbb{Z}$

31) $|-7.2| =$

[A] -7.2 [B] 7.2 [C] ± 7.2 [D] -9

32) $|0.14 - \pi| =$

[A] -3 [B] 3.14 [C] ± 3 [D] 3

33) The distance between the real numbers $-5,\ 6$ is

[A] -11 [B] 11 [C] -1 [D] 1

34) The distance between the real numbers $\frac{15}{8},\ \frac{23}{12}$ is

[A] $-\frac{1}{24}$ [B] $\pm\frac{1}{24}$ [C] $\frac{1}{12}$ [D] $\frac{1}{24}$

35) The distance between the points $(-2, -5)$ and $(3, 1)$ is

[A] $\sqrt{61}$ [B] $\sqrt{15}$ [C] $\sqrt{37}$ [D] $\sqrt{11}$

36) The solution of $|3x + 2| = 11$ is

[A] $-\frac{3}{13}, \frac{1}{3}$ [B] $-\frac{13}{3}, 3$ [C] $-\frac{13}{3}$ [D] 3

37) The solution of $|8x - 3| - 2 = 5$ is

[A] $-\frac{1}{2}, \frac{5}{4}$ [B] $\frac{1}{2}, \frac{5}{4}$ [C] $-\frac{1}{2}$ [D] $\frac{5}{4}$

38) The solution of $-2 < 2x + 1 \leq 6$ is

A $\left[-\frac{3}{2},\frac{5}{2}\right)$ B $(-3,4]$ C $\left(-\frac{1}{2},\frac{7}{2}\right]$ D $\left(-\frac{3}{2},\frac{5}{2}\right]$

39) The solution of $|x - 4| \leq 12$ is

A $(-\infty,-8]\cup[16,\infty)$ B $[-8,16]$ C $(-\infty,-16]\cup[8,\infty)$ D $(-8,16)$

40) The solution of $|x + 6| \geq 5$ is

A $(-\infty,-11]\cup[-1,\infty)$ B $[-11,1]$ C $(-\infty,1]\cup[11,\infty)$ D $(1,11)$

41) The solution of $|x - 3| > 5$ is

A $(-\infty,-2]\cup[8,\infty)$ B $[-2,8]$ C $(-\infty,-2)\cup(8,\infty)$ D $(-2,8)$

42) The solution of $|x + 4| = 3x - 8$ is

A 1 B $-6,-1$ C $1,6$ D 6

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